

KASNEB

CIFA PART III SECTION 6

ADVANCED PORTFOLIO MANAGEMENT

THURSDAY: 24 November 2016.

Time Allowed: 3 hours.

Answer ALL questions. Marks allocated to each question are shown at the end of the question. Show ALL your workings.

QUESTION ONE

- (a) Enumerate three roles that could be played by brokers and dealers when executing portfolio decisions in the financial market. (3 marks)
- (b) Financial markets are organised to provide liquidity, transparency, and assurance of completion, so that they might be judged by the degree to which they have these qualities in practice.

Required:

In the context of the above statement:

- (i) Examine three characteristics of a liquid market. (3 marks)
- (ii) Discuss two factors that could contribute to market illiquidity. (2 marks)
- (c) Jambolee Ltd. is a listed company on MSE, a local securities exchange in your country. On Monday, 10 November 2016 at 11.42 a.m. James Munene, a trader, sold 150 shares of Jambolee Ltd. at a price of Sh.813.30 per share. The table below encompasses all trades for Jambolee Ltd.'s share for that day:

| Time | Trade price (Sh.) | Volume of shares traded |
|----------|-------------------|-------------------------|
| 8.30 AM | 807.30 | 450 |
| 8.48 AM | 808.08 | 300 |
| 11.28 AM | 815.42 | 900 |
| 11.42 AM | 813.30 | 150 |
| 11.58 AM | 815.16 | 450 |
| 12.58 PM | 818.76 | 750 |
| 2.59 PM | 809.01 | 450 |

James Munene is evaluating the implicit costs of the trade, putting his main focus on the bid-ask spread and market impact using specified price benchmarks.

Required:

The estimated implicit transaction costs using each of the following price benchmarks:

- (i) Opening price. (2 marks)
- (ii) Closing price. (2 marks)
- (iii) Volume-weighted average price (VWAP). (5 marks)
- (d) Assume that the spread between a Ugandan bond and a Tanzanian bond is 300 basis points. This spread provides Tanzanian investors who purchase Ugandan bonds with an additional yield income of 75 basis points per quarter. The duration of the Tanzanian bonds is 8.3.

Required:

Determine the decline in the interest rate that would be needed to completely wipe out the quarterly yield advantage for the Tanzanian investor assuming that the Tanzanian interest rate declines. (3 marks)

(Total: 20 marks)

QUESTION TWO

- (a) In the context of portfolio management, discuss the following portfolio rebalancing strategies:

- (i) Calender rebalancing. (2 marks)
- (ii) Percentage-of-portfolio rebalancing. (2 marks)

- (b) Julius Poipoi, a self employed person who receives his income irregularly, has an investment account with Centrium Investment Firm. On 1 June 2016, he had a balance of Sh.150,000 in his account. On 13 June 2016, he deposited Sh.4,500 into his account. Further to that, on 20 June 2016, he deposited Sh.3,750.

His account was valued at Sh.157,500 and Sh.162,000 after the first and second contribution respectively. At the end of June 2016, his account was valued at Sh.165,000.

Required:

The time-weighted rate of return for the investment. (4 marks)

- (c) Andrew Koech, a portfolio manager with Smith Capital, wishes to increase the beta for one of the portfolios he manages from 0.9405 to 1.188 for a three-month period. The current market value of the portfolio under consideration is Sh.173,250,000. Smith Capital contemplates to use a futures contract priced at Sh.104,732.10 so as to adjust the portfolio beta. The beta of the futures contract is 0.9702.

Required:

(i) The number of futures contracts that should be bought or sold to achieve an increase in the portfolio beta of 1.188. (2 marks)

(ii) The value of the overall position and the effective beta of the portfolio assuming that the overall equity market goes up by 5.445% at the end of three months, while the stock portfolio under management also rises by 5.049% and that the futures contract is priced at Sh.110,385. (4 marks)

- (d) Ivy Kigen is considering an investment in distressed debt. She uses a 3-year horizon for evaluating the investment. Ivy Kigen analyses Jumia Capital, a newly distressed debt hedge fund and notes the following:

1. The annual management fee based on average net asset value (NAV) is 1%.
2. The performance fee paid monthly and calculated based on the monthly change in NAV, subject to a high water mark provision is 15%.
3. The lock up period is 3 years.
4. 14% of NAV is invested in a distressed airline company that recently filed for bankruptcy protection.
5. Net asset value per unit at the end of May 2016 was a new all time high of Sh.3,100.

Jumia Capital's subsequent month end NAV per unit was Sh.3,260 in June 2016, Sh.2,900 in July 2016 and Sh.3,140 in August 2016. There were no interim cash flows from clients during this three-month period.

Ivy Kigen learns of a competing distressed debt hedge fund with a similar performance fee and expected return, but only 1-year lock up period. She contacts Jumia Capital and states that she is considering investing in the competitor's fund. A representative for Jumia Capital replies that its 3-year lock up period is likely to be more favourable to Ivy Kigen than the competitors 1-year lock up period.

Required:

(i) The performance fee (in Sh. per unit) for the three months from June 2016 to August 2016. (2 marks)

(ii) Explain why Jumia Capital is subject to J-factor risk. (2 marks)

(iii) Support the representative's reply about Jumia Capital's lock-up period. (2 marks)

(Total: 20 marks)

QUESTION THREE

(a) Explain three reasons why an investor would consider investing in an indexed portfolio. (3 marks)

(b) Examine three approaches that an investment analyst could use to hedge against currency risk. (3 marks)

(c) An investor decides to pursue a contingent immunisation strategy over a 3-year time horizon. The investor has Sh.20 million to invest. The available 3-year immunisation rate is 4% and the investor will accept a minimum safety net return of 3.2%.

Required:

The initial surplus amount for the investor. (3 marks)

(d) George Orengo is an investment analyst at an asset management firm. Each year, he provides his firm with a report that includes a series of market forecasts. As part of his report, he uses the Grinold-Kroner model to forecast the expected rate of return on equities for the next 10 years. He uses the data below to prepare his forecast:

| Factor | 10-year forecast (annualised) (%) |
|--|-----------------------------------|
| • Dividend yield | 1.80 |
| • Dividend growth rate | 4.00 |
| • Change in price to earnings (P/E) multiple | 0.50 |
| • Inflation rate | 1.20 |
| • Change in the number of shares outstanding | -0.30 |
| • Real total earnings growth rate | 2.50 |

Required:

Using Grinold-Kroner model, calculate the following sources of return for equities:

- (i) Expected nominal earnings growth return. (1 mark)
- (ii) Expected repricing return. (1 mark)
- (iii) Expected income return. (1 mark)

(e) Simon Mbatia, a fixed income portfolio manager, manages a domestic bond fund. He is contemplating whether to purchase a 5-year callable, BBB rated corporate bond for the fund. The corporate bond's current yield is 4.90%. Simon Mbatia intends to use the risk-premium approach to decide on whether to purchase the bond for the fund. The trailing 12-month inflation rate is 1.10% and it is expected to remain constant at 1.50% per annum for the next five years. It is assumed that the illiquidity discount and tax premium are both zero.

The following information relates to the domestic bond market data:

1. Real risk-free interest rate is 1.30%
2. 1-year BBB rated credit risk spread (Over Treasuries) is 30 basis points.
3. 5-year BBB rated credit risk spread (Over Treasuries) is 80 basis points.
4. Spread of 5-year Treasury over 1-year Treasury is 100 basis points.
5. 1-year call risk spread is 20 basis points.
6. 5-year call risk spread is 60 basis points.

Required:

Based on the risk-premium approach, advise whether Simon Mbatia should purchase the corporate bond. (4 marks)

(f) Anthony Wekesa, a fixed income portfolio manager intends to add another bond to his portfolio. He uses mean-reversion analysis to determine the bond to purchase among the three bonds identified below:

| Bond | Credit spread and standard deviation in basis points (bps) | | |
|------|--|------------------------|------------------------------|
| | Current Spread | Historical mean spread | Standard deviation of spread |
| A | 300 | 210 | 50 |
| B | 320 | 230 | 30 |
| C | 340 | 240 | 40 |

The three bonds proposed above have similar durations, their credit spreads are normally distributed, and no structural changes are expected in the market.

Required:

Advise the portfolio manager on the most appropriate bond to purchase using the mean-reversion analysis approach.

(4 marks)

(Total: 20 marks)

QUESTION FOUR

(a) Assess how the following behavioural factors could influence asset allocation policy:

- (i) Loss aversion. (2 marks)
- (ii) Mental accounting. (2 marks)

(b) The following information relates to micro attribution analysis for a portfolio manager:

| Sector | Portfolio weight (%) | Benchmark weight (%) | Portfolio return (%) | Benchmark return (%) |
|-----------------|----------------------|----------------------|----------------------|----------------------|
| Agriculture | 21.53 | 28.70 | 9.47 | 4.16 |
| Energy | 34.91 | 45.44 | 8.21 | 5.43 |
| Financial | 31.35 | 11.79 | 6.82 | 4.98 |
| Technology | 12.21 | 14.07 | -9.02 | -1.71 |
| Total portfolio | 100.00 | 100.00 | 5.94 | 4.01 |

The manager's objective is to outperform the benchmark through superior security selection.

Required:

Calculate each of the following returns for the portfolio manager:

- (i) Pure-sector allocation return for the financial sector. (2 marks)
- (ii) Within-sector selection return for the technology sector. (2 marks)

(c) Anderson Mwadima is the portfolio manager for the Sh.200 million Natural Industries defined benefit pension fund. He is planning to make a presentation to the trustees of the pension plan. His firm has come up with long term capital market expectations as shown below:

| Asset class | Expected return (%) | Expected standard deviation (%) | Correlations | | | | | |
|-------------------------|---------------------|---------------------------------|--------------|------|------|------|------|--|
| | | | 1 | 2 | 3 | 4 | 5 | |
| Domestic equity | 12.00 | 16.00 | 1.00 | | | | | |
| Domestic bonds | 8.25 | 6.50 | 0.32 | 1.00 | | | | |
| International equity | 14.00 | 18.00 | 0.46 | 0.22 | 1.00 | | | |
| International bonds | 9.25 | 12.25 | 0.23 | 0.56 | 0.32 | 1.00 | | |
| Alternative investments | 11.50 | 21.00 | 0.25 | 0.11 | 0.08 | 0.06 | 1.00 | |

Using the capital market expectations, the portfolio manager identifies an efficient frontier with six corner portfolios with the characteristics shown below:

| Corner portfolio | Expected return (%) | Expected standard deviation (%) | Sharpe ratio | Asset class weights | | | | |
|------------------|---------------------|---------------------------------|--------------|---------------------|--------------------|--------------------------|-------------------------|-----------------------------|
| | | | | Domestic equity (%) | Domestic bonds (%) | International equity (%) | International bonds (%) | Alternative investments (%) |
| 1 | 14.00 | 18.00 | 0.639 | 0.00 | 0.00 | 100.00 | 0.00 | 0.00 |
| 2 | 13.66 | 16.03 | 0.696 | 0.00 | 0.00 | 85.36 | 0.00 | 14.64 |
| 3 | 13.02 | 13.58 | 0.775 | 21.69 | 0.00 | 56.56 | 0.00 | 21.75 |
| 4 | 12.79 | 13.00 | 0.792 | 21.48 | 0.00 | 52.01 | 5.24 | 21.27 |
| 5 | 10.54 | 8.14 | 0.988 | 9.40 | 51.30 | 26.55 | 0.00 | 12.76 |
| 6 | 8.70 | 6.32 | 0.981 | 0.00 | 89.65 | 4.67 | 0.00 | 5.68 |

Additional information:

1. The trustees have established a spending rate of 8.50%. Inflation is expected to be 2% per annum and the cost of managing the fund is expected to be 0.40%. The trustees would like to preserve the purchasing power of the fund and are concerned with multi period compounding issues.
2. The majority of plan participants are young, therefore additional liquidity needs are minimal.
3. The trustees would like to limit risk as defined by standard deviation to not more than 10% per year.

Required:

- (i) The fund's required rate of return. (2 marks)
- (ii) The appropriate strategic asset allocation for each asset class. (4 marks)
- (iii) The Sharpe ratio of the market and the risk-free rate. (4 marks)
- (iv) Advise on whether a risk-free asset should be included as an asset. (2 marks)

(Total: 20 marks)

QUESTION FIVE

(a) In relation to equity portfolio management, evaluate five advantages of short-extension strategies. (5 marks)

(b) Anthony Kioko is a small-cap growth manager who invests in domestic equities. He was hired by a pension fund that benchmarks him against a broad domestic market index provided below:

- Manager's return 18.0%
- Broad market return 15.0%
- Normal portfolio return 20.0%
- Total active risk 5.0%
- Misfit active risk 3.5%

Required:

(i) The true active risk. (3 marks)

(ii) Determine the manager's information ratio. (2 marks)

(c) Sukuk is a hedge fund that uses derivatives in its portfolio. A financial analyst is reviewing Sukuk's credit risk exposure. The firm's policy is to use a different counterparty for each derivative holding to limit its credit exposure to any single counterparty. Its current derivative holdings are shown below:

| Holding | Description | Notional principal Sh. | Current value Sh. |
|----------------------|---|------------------------|-------------------|
| • Interest rate swap | 1-year, quarterly payments; pay floating, receive fixed | 2,000,000 | 56,000 |
| • Forward contract | 2 years long natural gas | 5,000,000 | -225,000 |
| • Option | 6 months; long call option on domestic equity index | 5,000,000 | 487,000 |

All derivatives are traded over the counter (OTC) and are not subject to collateral requirements.

Required:

The hedge fund's total amount at risk of credit loss from its derivatives under its current policy. (4 marks)

(d) Paul Ng'ang'a, a portfolio manager for Arab Energy's European technology fund is concerned about currency fluctuations related to the equity portfolio. The portfolio is valued in United States Dollars (USD) but has exposure to multiple European currencies, primarily the Euro (EUR).

The portfolio manager formulates the following market expectations for the coming year:

- Expected return (in EUR) of the portfolio is 13.2%
- Standard deviation (in EUR) of the portfolio is 15%
- Expected USD/EUR spot rate in one year is 1 EUR = 1.2045 USD
- Standard deviation of the USD/EUR exchange rate is 5%
- Correlation between the USD/EUR exchange rate and the portfolio in (EUR) is -0.07

The market quotes indicated below are available from a currency dealer:

- USD/EUR spot rate 1.1930
- 1-year USD/EUR forward rate (bid-offer) 1.2065 – 1.2090

Paul Ng'ang'a is contemplating selling EUR and buying USD using a 1-year forward contract to fully hedge the EUR currency risk. He will execute the trade if he can achieve the following risk-return objectives:

Objective 1: Increase the portfolio's expected return (in USD) by at least 25 basis points (bps).

Objective 2: Reduce the portfolio's expected standard deviation in USD by at least 30 basis points (bps).

Required:

Based on Paul Ng'ang'a's market expectations, determine whether he should execute the formal trade with respect to each of the following risk-return objectives:

(i) Objective 1. (3 marks)

(ii) Objective 2. (3 marks)

(Total: 20 marks)

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Present Value of 1 Received at the End of n Periods:

$$PVIF_{r,n} = 1/(1+r)^n = (1+r)^{-n}$$

| Period | 1% | 2% | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% | 12% | 14% | 15% | 16% | 18% | 20% | 24% | 28% | 32% | 36% |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | .9901 | .9804 | .9709 | .9615 | .9524 | .9434 | .9346 | .9259 | .9174 | .9091 | .8929 | .8772 | .8696 | .8621 | .8475 | .8333 | .8065 | .7813 | .7576 | .7353 |
| 2 | .9803 | .9612 | .9426 | .9246 | .9070 | .8900 | .8734 | .8573 | .8417 | .8264 | .7972 | .7695 | .7561 | .7432 | .7182 | .6944 | .6504 | .6104 | .5739 | .5407 |
| 3 | .9706 | .9423 | .9151 | .8890 | .8638 | .8396 | .8163 | .7938 | .7722 | .7513 | .7118 | .6750 | .6575 | .6407 | .6086 | .5787 | .5245 | .4768 | .4348 | .3975 |
| 4 | .9610 | .9238 | .8885 | .8548 | .8227 | .7921 | .7629 | .7350 | .7084 | .6830 | .6355 | .5921 | .5718 | .5523 | .5158 | .4823 | .4230 | .3725 | .3294 | .2923 |
| 5 | .9515 | .9057 | .8626 | .8219 | .7835 | .7473 | .7130 | .6806 | .6499 | .6209 | .5674 | .5194 | .4972 | .4761 | .4371 | .4019 | .3411 | .2910 | .2495 | .2149 |
| 6 | .9420 | .8880 | .8375 | .7903 | .7462 | .7050 | .6663 | .6302 | .5963 | .5645 | .5066 | .4556 | .4323 | .4104 | .3704 | .3349 | .2751 | .2274 | .1890 | .1580 |
| 7 | .9327 | .8706 | .8131 | .7599 | .7107 | .6651 | .6227 | .5835 | .5470 | .5132 | .4523 | .3996 | .3759 | .3538 | .3139 | .2791 | .2218 | .1776 | .1432 | .1162 |
| 8 | .9235 | .8535 | .7894 | .7307 | .6768 | .6274 | .5820 | .5403 | .5019 | .4665 | .4039 | .3506 | .3269 | .3050 | .2660 | .2326 | .1789 | .1388 | .1085 | .0854 |
| 9 | .9143 | .8368 | .7664 | .7026 | .6446 | .5919 | .5439 | .5002 | .4604 | .4241 | .3606 | .3075 | .2843 | .2630 | .2255 | .1938 | .1443 | .1084 | .0822 | .0628 |
| 10 | .9053 | .8203 | .7441 | .6756 | .6139 | .5584 | .5083 | .4632 | .4224 | .3855 | .3220 | .2697 | .2472 | .2267 | .1911 | .1615 | .1164 | .0847 | .0623 | .0462 |
| 11 | .8963 | .8043 | .7224 | .6496 | .5847 | .5268 | .4751 | .4289 | .3875 | .3505 | .2875 | .2366 | .2149 | .1954 | .1619 | .1346 | .0938 | .0662 | .0472 | .0340 |
| 12 | .8874 | .7885 | .7014 | .6246 | .5568 | .4970 | .4440 | .3971 | .3555 | .3186 | .2567 | .2076 | .1869 | .1685 | .1372 | .1122 | .0757 | .0517 | .0357 | .0250 |
| 13 | .8787 | .7730 | .6810 | .6006 | .5303 | .4688 | .4150 | .3677 | .3262 | .2897 | .2292 | .1821 | .1625 | .1452 | .1163 | .0935 | .0610 | .0404 | .0271 | .0184 |
| 14 | .8700 | .7579 | .6611 | .5775 | .5051 | .4423 | .3878 | .3405 | .2992 | .2633 | .2046 | .1597 | .1413 | .1252 | .0985 | .0779 | .0492 | .0316 | .0205 | .0135 |
| 15 | .8613 | .7430 | .6419 | .5553 | .4810 | .4173 | .3624 | .3152 | .2745 | .2394 | .1827 | .1401 | .1229 | .1079 | .0835 | .0649 | .0397 | .0247 | .0155 | .0099 |
| 16 | .8528 | .7284 | .6232 | .5339 | .4581 | .3936 | .3387 | .2919 | .2519 | .2176 | .1631 | .1229 | .1069 | .0930 | .0708 | .0541 | .0320 | .0193 | .0118 | .0073 |
| 17 | .8444 | .7142 | .6050 | .5134 | .4363 | .3714 | .3166 | .2703 | .2311 | .1978 | .1456 | .1078 | .0929 | .0802 | .0600 | .0451 | .0258 | .0150 | .0089 | .0054 |
| 18 | .8360 | .7002 | .5874 | .4936 | .4155 | .3503 | .2959 | .2502 | .2120 | .1799 | .1300 | .0946 | .0808 | .0691 | .0508 | .0376 | .0208 | .0118 | .0068 | .0039 |
| 19 | .8277 | .6864 | .5703 | .4746 | .3957 | .3305 | .2765 | .2317 | .1945 | .1635 | .1161 | .0829 | .0703 | .0596 | .0431 | .0313 | .0168 | .0092 | .0051 | .0029 |
| 20 | .8195 | .6730 | .5537 | .4564 | .3769 | .3118 | .2584 | .2145 | .1784 | .1486 | .1037 | .0728 | .0611 | .0514 | .0365 | .0261 | .0135 | .0072 | .0039 | .0021 |
| 25 | .7798 | .6095 | .4776 | .3751 | .2953 | .2330 | .1842 | .1460 | .1160 | .0923 | .0588 | .0378 | .0304 | .0245 | .0160 | .0105 | .0046 | .0021 | .0010 | .0005 |
| 30 | .7419 | .5521 | .4120 | .3083 | .2314 | .1741 | .1314 | .0994 | .0754 | .0573 | .0334 | .0196 | .0151 | .0116 | .0070 | .0042 | .0016 | .0006 | .0002 | .0001 |
| 40 | .6717 | .4529 | .3066 | .2083 | .1420 | .0972 | .0668 | .0460 | .0318 | .0221 | .0107 | .0053 | .0037 | .0026 | .0013 | .0007 | .0002 | .0001 | . | . |
| 50 | .6080 | .3715 | .2281 | .1407 | .0872 | .0543 | .0339 | .0213 | .0134 | .0085 | .0035 | .0014 | .0009 | .0006 | .0003 | .0001 | . | . | . | . |
| 60 | .5504 | .3048 | .1697 | .0951 | .0535 | .0303 | .0173 | .0099 | .0057 | .0033 | .0011 | .0004 | .0002 | .0001 | . | . | . | . | . | . |

* The factor is zero to four decimal places

Present Value of an Annuity of 1 Per Period for n Periods:

$$PVIF_{r,t} = \sum_{t=1}^n \frac{1}{(1+r)^t} = \frac{1 - \frac{1}{(1+r)^n}}{r}$$

| Number of payments | 1% | 2% | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% | 12% | 14% | 15% | 16% | 18% | 20% | 24% | 28% | 32% |
|--------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 | 0.9901 | 0.9804 | 0.9709 | 0.9615 | 0.9524 | 0.9434 | 0.9346 | 0.9259 | 0.9174 | 0.9091 | 0.8929 | 0.8772 | 0.8696 | 0.8621 | 0.8475 | 0.8333 | 0.8065 | 0.7813 | 0.7576 |
| 2 | 1.9704 | 1.9416 | 1.9135 | 1.8861 | 1.8594 | 1.8334 | 1.8080 | 1.7833 | 1.7591 | 1.7355 | 1.6901 | 1.6467 | 1.6257 | 1.6052 | 1.5656 | 1.5278 | 1.4568 | 1.3916 | 1.3315 |
| 3 | 2.9410 | 2.8839 | 2.8286 | 2.7751 | 2.7232 | 2.6730 | 2.6243 | 2.5771 | 2.5313 | 2.4869 | 2.4018 | 2.3216 | 2.2832 | 2.2459 | 2.1743 | 2.1065 | 1.9813 | 1.8684 | 1.7663 |
| 4 | 3.9020 | 3.8077 | 3.7171 | 3.6299 | 3.5460 | 3.4651 | 3.3872 | 3.3121 | 3.2397 | 3.1699 | 3.0373 | 2.9137 | 2.8550 | 2.7982 | 2.6901 | 2.5887 | 2.4043 | 2.2410 | 2.0957 |
| 5 | 4.8534 | 4.7135 | 4.5797 | 4.4518 | 4.3295 | 4.2124 | 4.1002 | 3.9927 | 3.8897 | 3.7908 | 3.6048 | 3.4331 | 3.3522 | 3.2743 | 3.1272 | 2.9906 | 2.7454 | 2.5320 | 2.3452 |
| 6 | 5.7955 | 5.6014 | 5.4172 | 5.2421 | 5.0757 | 4.9173 | 4.7665 | 4.6229 | 4.4859 | 4.3553 | 4.1114 | 3.8887 | 3.7845 | 3.6847 | 3.4976 | 3.3255 | 3.0205 | 2.7594 | 2.5342 |
| 7 | 6.7282 | 6.4720 | 6.2303 | 6.0021 | 5.7864 | 5.5824 | 5.3893 | 5.2064 | 5.0330 | 4.8684 | 4.5638 | 4.2883 | 4.1604 | 4.0386 | 3.8115 | 3.6046 | 3.2423 | 2.9370 | 2.6775 |
| 8 | 7.6517 | 7.3255 | 7.0197 | 6.7327 | 6.4632 | 6.2098 | 5.9713 | 5.7466 | 5.5349 | 5.3349 | 4.9676 | 4.6389 | 4.4873 | 4.3436 | 4.0776 | 3.8372 | 3.4212 | 3.0758 | 2.7860 |
| 9 | 8.5660 | 8.1622 | 7.7861 | 7.4353 | 7.1078 | 6.8017 | 6.5152 | 6.2469 | 5.9952 | 5.7590 | 5.3282 | 4.9464 | 4.7716 | 4.6065 | 4.3030 | 4.0310 | 3.5655 | 3.1842 | 2.8681 |
| 10 | 9.4713 | 8.9826 | 8.5302 | 8.1109 | 7.7217 | 7.3601 | 7.0236 | 6.7101 | 6.4177 | 6.1446 | 5.6502 | 5.2161 | 5.0188 | 4.8332 | 4.4941 | 4.1925 | 3.6819 | 3.2689 | 2.9304 |
| 11 | 10.3676 | 9.7868 | 9.2526 | 8.7605 | 8.3064 | 7.8869 | 7.4987 | 7.1390 | 6.8052 | 6.4951 | 5.9377 | 5.4527 | 5.2337 | 5.0286 | 4.6560 | 4.3271 | 3.7757 | 3.3351 | 2.9776 |
| 12 | 11.2551 | 10.5753 | 9.9540 | 9.3851 | 8.8633 | 8.3838 | 7.9427 | 7.5361 | 7.1607 | 6.8137 | 6.1944 | 5.6603 | 5.4206 | 5.1971 | 4.7932 | 4.4392 | 3.8514 | 3.3868 | 3.0133 |
| 13 | 12.1337 | 11.3484 | 10.6350 | 9.9856 | 9.3936 | 8.8527 | 8.3577 | 7.9038 | 7.4869 | 7.1034 | 6.4235 | 5.8424 | 5.5831 | 5.3423 | 4.9095 | 4.5327 | 3.9124 | 3.4272 | 3.0404 |
| 14 | 13.0037 | 12.1062 | 11.2961 | 10.5631 | 9.8986 | 9.2950 | 8.7455 | 8.2442 | 7.7862 | 7.3667 | 6.6282 | 6.0021 | 5.7245 | 5.4675 | 5.0081 | 4.6106 | 3.9616 | 3.4587 | 3.0609 |
| 15 | 13.8651 | 12.8493 | 11.9379 | 11.1184 | 10.3797 | 9.7122 | 9.1079 | 8.5595 | 8.0607 | 7.6061 | 6.8109 | 6.1422 | 5.8474 | 5.5755 | 5.0916 | 4.6755 | 4.0013 | 3.4834 | 3.0764 |
| 16 | 14.7179 | 13.5777 | 12.5611 | 11.6523 | 10.8378 | 10.1059 | 9.4466 | 8.8514 | 8.3126 | 7.8237 | 6.9740 | 6.2651 | 5.9542 | 5.6685 | 5.1624 | 4.7296 | 4.0333 | 3.5026 | 3.0882 |
| 17 | 15.5623 | 14.2919 | 13.1661 | 12.1657 | 11.2741 | 10.4773 | 9.7632 | 9.1216 | 8.5436 | 8.0216 | 7.1196 | 6.3729 | 6.0472 | 5.7487 | 5.2223 | 4.7746 | 4.0591 | 3.5177 | 3.0971 |
| 18 | 16.3983 | 14.9920 | 13.7535 | 12.6593 | 11.6896 | 10.8276 | 10.0591 | 9.3719 | 8.7556 | 8.2014 | 7.2497 | 6.4674 | 6.1280 | 5.8178 | 5.2732 | 4.8122 | 4.0799 | 3.5294 | 3.1039 |
| 19 | 17.2260 | 15.6785 | 14.3238 | 13.1339 | 12.0853 | 11.1581 | 10.3356 | 9.6036 | 8.9501 | 8.3649 | 7.3658 | 6.5504 | 6.1982 | 5.8775 | 5.3162 | 4.8435 | 4.0967 | 3.5386 | 3.1090 |
| 20 | 18.0456 | 16.3514 | 14.8775 | 13.5903 | 12.4622 | 11.4699 | 10.5940 | 9.8181 | 9.1285 | 8.5136 | 7.4694 | 6.6231 | 6.2593 | 5.9288 | 5.3527 | 4.8696 | 4.1103 | 3.5458 | 3.1129 |
| 25 | 22.0232 | 19.5235 | 17.4131 | 15.6221 | 14.0939 | 12.7834 | 11.6536 | 10.6748 | 9.8226 | 9.0770 | 7.8431 | 6.8729 | 6.4641 | 6.0971 | 5.4669 | 4.9476 | 4.1474 | 3.5640 | 3.1220 |
| 30 | 25.8077 | 22.3965 | 19.6004 | 17.2920 | 15.3725 | 13.7648 | 12.4090 | 11.2578 | 10.2737 | 9.4269 | 8.0552 | 7.0027 | 6.5660 | 6.1772 | 5.5168 | 4.9789 | 4.1601 | 3.5693 | 3.1242 |
| 40 | 32.8347 | 27.3555 | 23.1148 | 19.7928 | 17.1591 | 15.0463 | 13.3317 | 11.9246 | 10.7574 | 9.7791 | 8.2438 | 7.1050 | 6.6418 | 6.2335 | 5.5482 | 4.9966 | 4.1659 | 3.5712 | 3.1250 |
| 50 | 39.1961 | 31.4236 | 25.7298 | 21.4822 | 18.2559 | 15.7619 | 13.8007 | 12.2335 | 10.9617 | 9.9148 | 8.3045 | 7.1327 | 6.6605 | 6.2463 | 5.5541 | 4.9995 | 4.1666 | 3.5714 | 3.1250 |
| 60 | 44.9550 | 34.7609 | 27.6756 | 22.6235 | 18.9293 | 16.1614 | 14.0392 | 12.3766 | 11.0480 | 9.9672 | 8.3240 | 7.1401 | 6.6651 | 6.2402 | 5.5553 | 4.9999 | 4.1667 | 3.5714 | 3.1250 |